

perpendicular gradients

- [SQA] 1. Find the equation of the perpendicular bisector of the line joining A(2, -1) and B(8, 3).

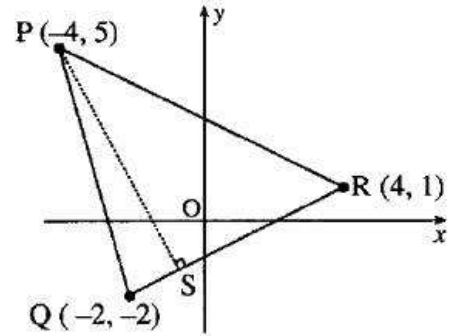
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Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	4	C	CN	G2, G5		1996 P1 Q1

<ul style="list-style-type: none"> •¹ midpoint = (5,1) •² $m_{AB} = \frac{2}{3}$ •³ $m_{\perp} = -\frac{3}{2}$ •⁴ $y - 1 = -\frac{3}{2}(x - 5)$
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- [SQA] 2. P(-4, 5), Q(-2, -2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P.

3



Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	CN	G2, G5, G3		1997 P1 Q1

<ul style="list-style-type: none"> •¹ $m_{QR} = \frac{1}{2}$ •² $m_{PN} = -2$ •³ $PN: y - 4 = -2(x + 3)$

- [SQA] 3. The vertices of a triangle are P(-1, 1), Q(2, 1) and R(-6, 2). Find the equation of the altitude of triangle PQR, drawn from P.

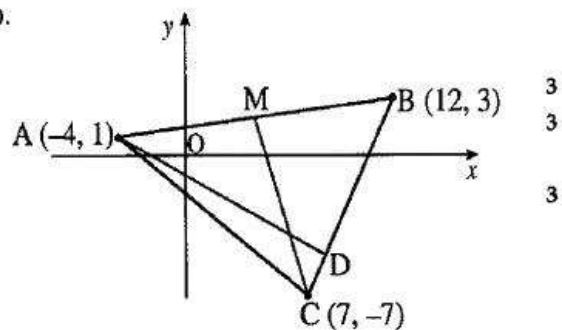
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Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	CN	G3, G5		1989 P1 Q1

<ul style="list-style-type: none"> •¹ $m_{QR} = -\frac{1}{8}$ •² $m_{\perp} = 8$ •³ $y - (-1) = 8(x - (-1))$

[SQA] 4. A triangle ABC has vertices A (-4, 1), B (12, 3) and C (7, -7).

- (a) Find the equation of the median CM.
 (b) Find the equation of the altitude AD.
 (c) Find the coordinates of the point of intersection of CM and AD.



Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	3	C	CN	G3		1999 P2 Q1
(b)	3	C	CN	G3, G5		
(c)	3	C	CN	G8		

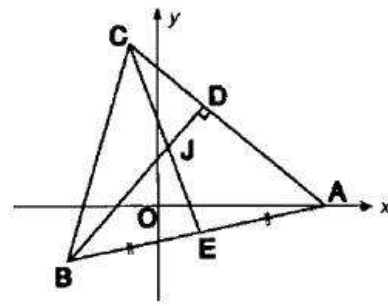
- (a) •¹ midpoint = (4, 2)
 •² $m_{MC} = -3$
 •³ $y - 2 = -3(x - 4)$ or $y - (-7) = -3(x - 7)$
- (b) •⁴ $m_{BC} = 2$
 •⁵ $m_{\perp} = -\frac{1}{2}$
 •⁶ $y - 1 = -\frac{1}{2}(x - (-4))$
- (c) •⁷ e.g. $3x + y = 14$ and $x + 2y = -2$
 •⁸ attempt to eliminate a variable
 •⁹ (6, -4)

[SQA]

5. In the diagram A is the point (7,0), B is (-3,-2) and C(-1,8).

The median CE and the altitude BD intersect at J.

- (a) Find the equations of CE and BD.
 (b) Find the co-ordinates of J.

6
2

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	6	C	NC	G3, G5, G8		1992 P1 Q2
(b)	2	C	NC	G8		

•¹ $E = (2, -1)$

•² $m_{CE} = -3$

•³ $y - (-1) = -3(x - 2)$ or $y - 8 = -3(x - (-1))$

•⁴ $m_{AC} = -1$

•⁵ $m_{BD} = -1$

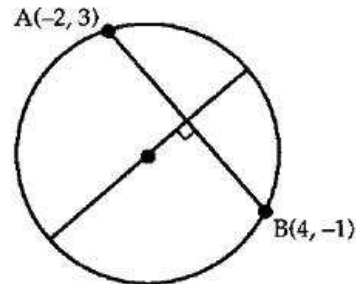
•⁶ $y - (-2) = 1(x - (-3))$

•⁷ strat: attempt to solve simultaneously

•⁸ $J = (1, 2)$

[SQA]

6. A circle passes through A(-2, 3) and B(4, -1). Find the equation of the perpendicular to the chord AB.



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Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	4	C	CN	G5, G3		1990 P1 Q3

•¹ midpt = (1, 1)

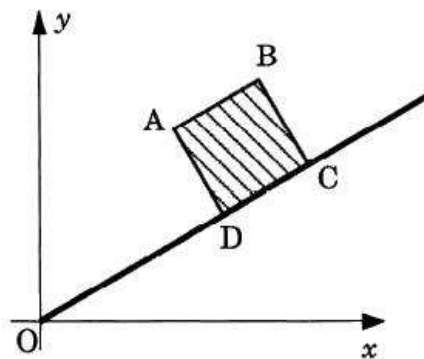
•² $m_{AB} = -\frac{2}{3}$

•³ $m_{diam} = \frac{3}{2}$

•⁴ $y - 1 = \frac{3}{2}(x - 1)$

[SQA]

7. ABCD is a square. A is the point with coordinates (3,4) and ODC has equation $y = \frac{1}{2}x$.



- (a) Find the equation of the line AD. (3)
 (b) Find the coordinates of D. (3)
 (c) Find the area of the square ABCD. (2)

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	3	C	CN	G5, G3		1994 P2 Q2
(b)	2	C	CN	G1		
(c)	2	C	CN	G1		

- (a) •¹ using $m_1 m_2 = -1$
 •² $m_{AD} = -2$
 •³ $y - 4 = -2(x - 3)$
- (b) •⁴ strategy for sim. equations
 •⁵ $2x + y = 10$ or equiv
 •⁶ (4, 2)
- (c) •⁷ strategy : find length of AD
 •⁸ 5

[SQA]

8. P, Q and R have coordinates $(1, -2)$, $(6, 3)$ and $(9, 14)$ respectively and are three vertices of a kite PQRS.

(a) Find the equations of the diagonals of this kite and the coordinates of the point where they intersect.

7

(b) Find the coordinates of the fourth vertex S.

2

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	7	C	CN	G8, G3, G5		1990 P2 Q2
(b)	2	C	CN	G8		

- (a)
- ¹ $m_{PR} = 2$
 - ² PR: e.g. $y + 2 = 2(x - 1)$
 - ³ knowing to use $m_1 m_2 = 1$ for m_{QS}
 - ⁴ $m_{QS} = -\frac{1}{2}$
 - ⁵ QS: e.g. $y - 3 = -\frac{1}{2}(x - 6)$
 - ⁶ knowing to solve simultaneously
 - ⁷ $S = (4, 4)$

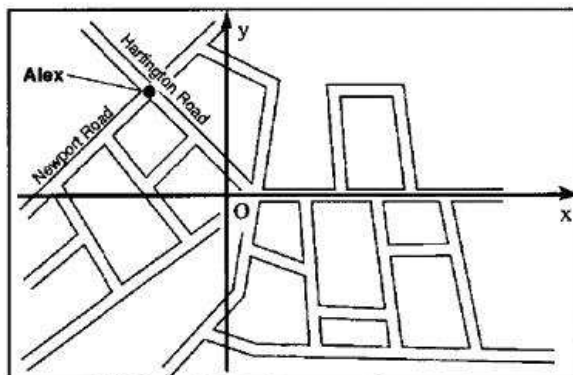
- (b)
- ⁸ $\vec{QM} = \vec{MS}$ or equivalent indication
 - ⁹ $S = (2, 5)$

[SQA]

9. Relative to the axes shown and with an appropriate scale, Alex stands at the point $(-2, 3)$ where Hartington Road meets Newport Road.

(a) Find the equation of Newport Road which is perpendicular to Hartington Road.

(b) Brenda is waiting for a bus at the point $(-5, 1)$. Show that Brenda is standing on Newport Road.



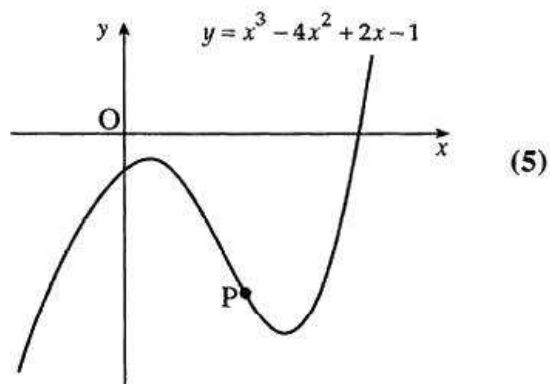
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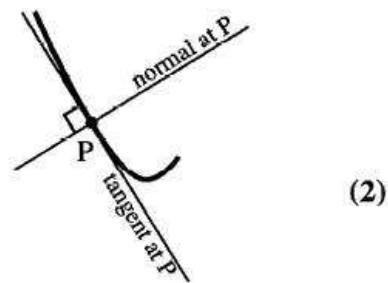
Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	3	C	CN	G3, G5		1993 P1 Q2
(b)	1	C	CN	A6		

- ¹ $m_{OA} = -\frac{3}{2}$
- ² $m_{\perp} = \frac{2}{3}$
- ³ $y - 3 = \frac{2}{3}(x + 2)$
- ⁴ verify that $(-5, 1)$ lies on this line

- [SQA] 10. (a) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the tangent to the curve at the point P where $x = 2$.



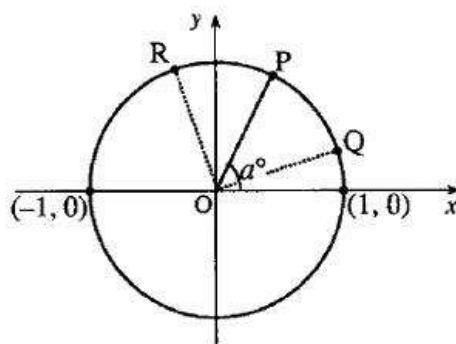
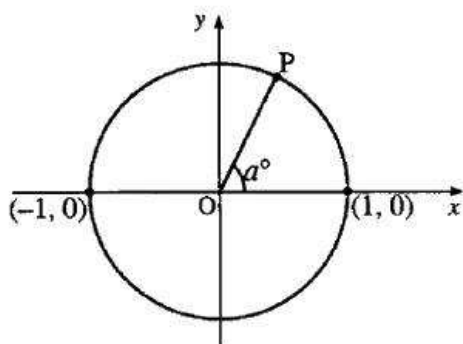
- (b) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P. Find the angle which the normal at P makes with the positive direction of the x-axis.



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	5	C	CN	G3, C4, G3		1998 P2 Q3
(b)	2	C	CN	G2, G5		

- (a)
- ¹ $\frac{dy}{dx} = \dots$
 - ² $3x^2 - 8x + 2$
 - ³ gradient = -2 (calculated from $\frac{dy}{dx}$)
 - ⁴ $y_A = -5$
 - ⁵ $y + 5 = -2(x - 2)$
- (b)
- ⁶ $m_{\text{normal}} = \frac{1}{2}$
 - ⁷ angle = $\tan^{-1} \frac{1}{2}$

- [SQA] 11. The diagram shows a circle of radius 1 unit and centre the origin. The radius OP makes an angle a° with the positive direction of the x -axis.



- (a) Show that P is the point $(\cos a^\circ, \sin a^\circ)$. 1
- (b) If $\widehat{POQ} = 45^\circ$, deduce the coordinates of Q in terms of a . 1
- (c) If $\widehat{POR} = 45^\circ$, deduce the coordinates of R in terms of a . 1
- (d) Hence find an expression for the gradient of QR in its simplest form. 4
- (e) Show that the tangent to the circle at P is parallel to QR. 2

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a-c)	3	C	NC	CGD		1999 P2 Q8
(d)	4	A/B	NC	T8, G2		
(e)	2	A/B	NC	G2, G5		

(a) •¹ proof e.g. showing rt-angled triangle with "1" and a°

(b) •² Q is $(\cos(a - 45)^\circ, \sin(a - 45)^\circ)$

(c) •³ R is $(\cos(a + 45)^\circ, \sin(a + 45)^\circ)$

(d) •⁴ $\frac{\sin(a+45) - \sin(a-45)}{\cos(a+45) - \cos(a-45)}$

•⁵ $\frac{\sin a \cos 45 + \cos a \sin 45 - \sin a \cos 45 + \cos a \sin 45}{\cos a \cos 45 - \sin a \sin 45 - \cos a \cos 45 - \sin a \sin 45}$

•⁶ $\frac{2 \cos a \sin 45}{-2 \sin a \sin 45}$

•⁷ $-\frac{1}{\tan a}$

(e) •⁸ $m_{OP} = \frac{\sin a}{\cos a} = \tan a$

•⁹ $m_{\text{tgt at P}} = -\frac{1}{\tan a}$

[SQA] 12.

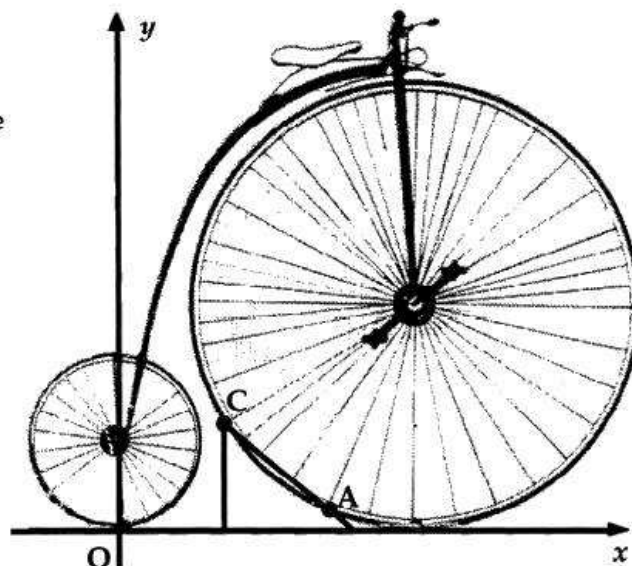
A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.

With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is

$$x^2 + y^2 - 6y = 0$$

and the equation of the front wheel is

$$x^2 + y^2 - 28x - 20y + 196 = 0.$$



- (a) (i) Find the distance between the centres of the two wheels.
 (ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. (8)
- (b) B(7,3) is half-way between A and C, and P is the centre of the front wheel.
 (i) Find the gradient of PB.
 (ii) Hence find the equation of AC and the coordinates of A and C. (8)

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	8	C	CR	G9, G1		1994 P2 Q4
(b)	8	C	CR	G2, G5, G12		

<p>(a)</p> <ul style="list-style-type: none"> •¹ centre (0, 3) •² centre (14, 10) •³ distance between centres = $\sqrt{245}$ •⁴ radius = 3 •⁵ radius = 10 •⁶ strategy (clearance = distance between centres minus sum of radii) •⁷ $\sqrt{245} - 13$ •⁸ 133 mm or equivalent 	<p>(b)</p> <ul style="list-style-type: none"> •⁹ $m_{PB} = 1$ •¹⁰ $m_{AC} = -1$ •¹¹ $y - 3 = -(x - 7)$ for AC •¹² strategy: substitute •¹³ substituting correctly •¹⁴ eg $2x^2 - 28x + 96 = 0$ •¹⁵ $x = 6, 8$ (or $y = 2, 4$) •¹⁶ (6, 4) and (8, 2)
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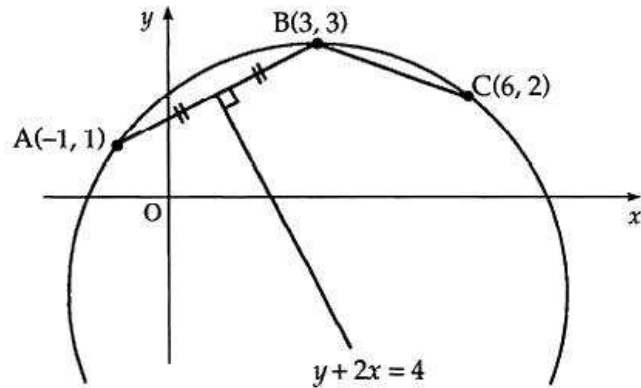
- [SQA] 13. Find the equation of the tangent at the point $(3,4)$ on the circle $x^2 + y^2 + 2x - 4y - 15 = 0$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	4	C	CN	G2, G5, G9		1996 P1 Q4

<ul style="list-style-type: none">•¹ centre = $(-1,2)$•² $m_{radius} = \frac{1}{2}$•³ $m_{tgr} = -2$•⁴ $y - 4 = -2(x - 3)$

- [SQA] 14. (a) In the diagram, A is the point $(-1, 1)$, B is $(3, 3)$ and C is $(6, 2)$. The perpendicular bisector of AB has equation $y + 2x = 4$. Find the equation of the perpendicular bisector of BC. (4)

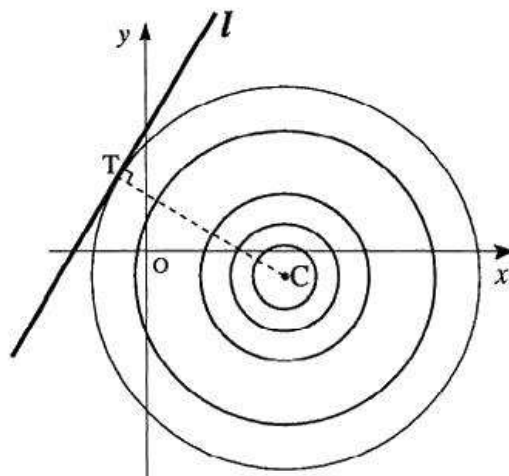


- (b) Find the centre and the equation of the circle which passes through A, B and C. (6)

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	4	C	CN	G5, G3		1991 P2 Q2
(b)	6	C	CN	G10, G1		

(a)	<ul style="list-style-type: none"> •¹ $m_{BC} = -\frac{1}{3}$ •² $m_{\perp} = 3$ •³ $\text{midpoint}_{BC} = \left(\frac{9}{2}, \frac{5}{2}\right)$ •⁴ $y - \frac{5}{2} = 3\left(x - \frac{9}{2}\right)$
(b)	<ul style="list-style-type: none"> •⁵ $y - 3x = -11$ •⁶ perp. bisector passes thr' centre stated explicitly •⁷ using $y - 3x = -11$ and $y + 2x = 4$ •⁸ $(3, -2)$ •⁹ $r^2 = 25$ •¹⁰ $(x - 3)^2 + (y + 2)^2 = 25$

- [SQA] 15. In an experiment with a ripple tank, a series of concentric circles with centre $C(4,-1)$ is formed as shown in the diagram.



The line l with equation $y = 2x + 6$ represents a barrier placed in the tank. The largest complete circle touches the barrier at the point T .

- (a) Find the equation of the radius CT . (3)
- (b) Find the equation of the largest complete circle. (5)

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	3	C	CN	G5, G3		1993 P2 Q3
(b)	5	C	CN	G10		

- (a)
- ¹ $m_l = 2$
 - ² $m_r = -\frac{1}{2}$
 - ³ $y + 1 = -\frac{1}{2}(x - 4)$
- (b)
- ⁴ $(x - 4)^2 + (y + 1)^2 = r^2$
 - ⁵ $(x - 4)^2 + (2x + 7)^2 = r^2$
 - ⁶ $5x^2 + 20x + (65 - r^2) = 0$
 - ⁷ $\Delta = 400 - 4 \times 5(65 - r^2) = 0$
 - ⁸ $r^2 = 45$

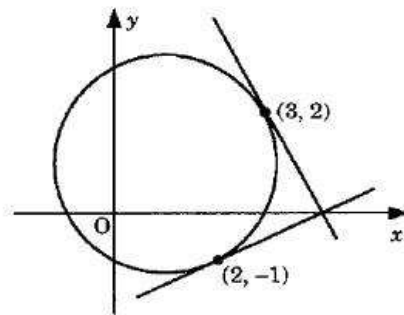
[SQA] 16. The circle shown in the diagram has equation

$$(x-1)^2 + (y-1)^2 = 5.$$

Tangents are drawn at the points $(3, 2)$ and $(2, -1)$.

Write down the coordinates of the centre of the circle and

hence show that the tangents are perpendicular to each other.



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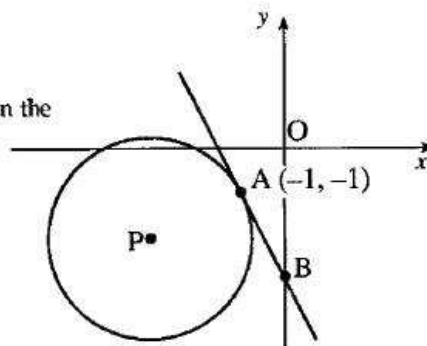
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	4	C	CN	G9, G5		1994 P1 Q5

<ul style="list-style-type: none"> •¹ centre = $(1, 1)$ •² $m_{\text{radii}} = \frac{1}{2}, -2$ •³ $m_{\text{tgts}} = -2, \frac{1}{2}$ •⁴ $-2 \times \frac{1}{2} = -1 \Rightarrow$ tgts are \perp 	OR	<ul style="list-style-type: none"> •¹ centre = $(1, 1)$ •² $r = \sqrt{5}, d = \sqrt{10}$ •³ Show $\hat{ACB} = 90^\circ$ •⁴ State tangents \perp to radii 	
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[SQA] 17. (a) The diagram shows a circle, centre P, with equation

$$x^2 + y^2 + 6x + 4y + 8 = 0.$$

Find the equation of the tangent at the point A(-1, -1) on the circle.



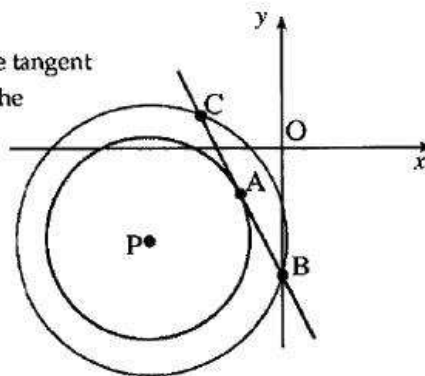
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- (b) The tangent crosses the y -axis at B.
Find the coordinates of B.

1

- (c) Another circle, centre P, is drawn passing through B. The tangent at A meets the second circle at the point C, as shown in the diagram.

Write down the coordinates of C.



1

- (d) Find the equation of the circle with BC as diameter.

2

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	4	C	CN	G9, G5		1999 P2 Q2
(b)	1	C	CN	A6		
(c)	1	C	CN	CGD		
(d)	2	C	CN	G10		

(a) •¹ centre = (-3, -2)

•² $m_{rad} = \frac{1}{2}$

•³ $m_{tgt} = -2$

•⁴ $y - (-1) = -2(x - (-1))$

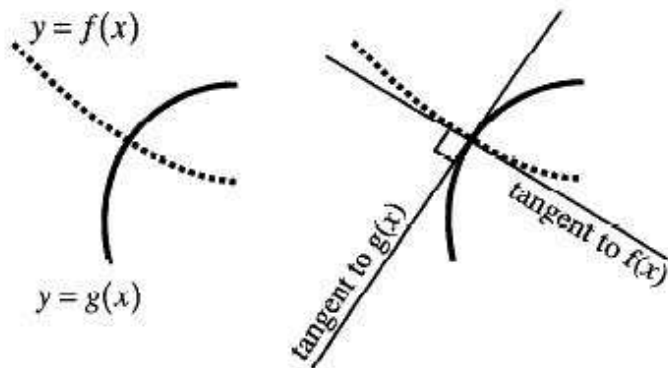
(b) •⁵ $B = (0, -3)$

(c) •⁶ $C = (-2, 1)$

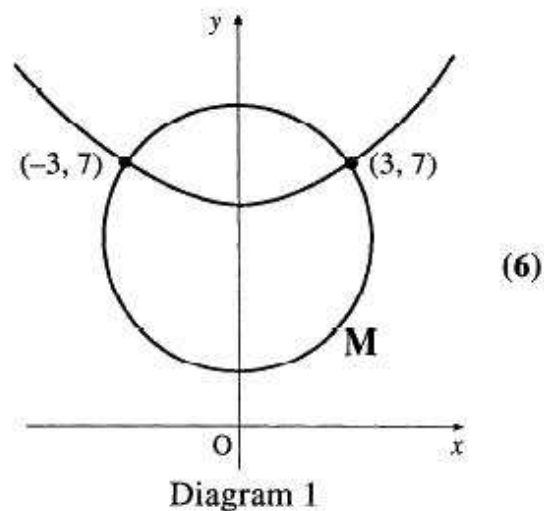
(d) •⁷ $r^2 = 5$

•⁸ $(x+1)^2 + (y+1)^2 = 5$

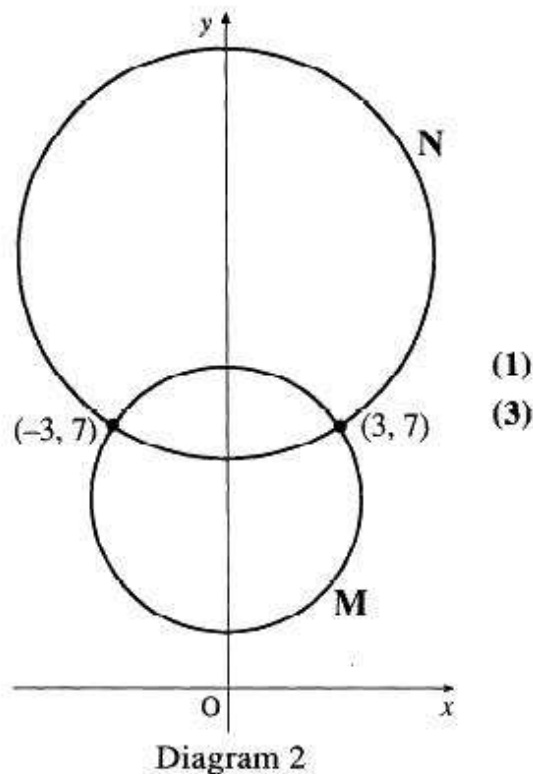
Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



- (a) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y - 5)^2 = 13$. These two curves intersect at $(3, 7)$ and $(-3, 7)$. Prove that these curves are orthogonal.



- (b) Diagram 2 shows the circle M, from (a) above, which is orthogonal to the circle N. The circles intersect at $(3, 7)$ and $(-3, 7)$.
- Write down the equation of the tangent to circle M at the point $(-3, 7)$.
 - Hence find the equation of circle N.



Part	Marks	Level	Calc.	Content	Answer	U2 OC4